

Brane Bounce from logarithmic entropic corrections in the bulk

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We calculate new corrections to the Brane-world dynamics, lying in a 5D Schwarzschild-De Sitter black hole, generalizing the result of Nojiri, Odintsov and Ogushi (NOO) in Ref.[1]. The NOO entropy effect is based on the Logharitmic correction to the bulk entropy firstly calculated by Mukherji and Pal in Ref.[2]. We calculate higher order contributions to the brane worldsheet. The extra terms obtained lead to interesting implications in brane-cosmology. In particular, new entropic terms rapidly disappear in the late Universe while exploding in the very Early Universe. In particular, we show that they may trigger a cosmological bounce in the very early Universe. On the other hand, they contribute to the cosmological expansion in the Late Universe. We also discuss a scenario in which the BLK anisotropies are washed-out, toward a new Ekpyrotic Brane Cosmology.

I. INTRODUCTION

The semiclassical analysis of Branes lying in a higher dimensional bulk is crucially important for our understanding of string theory and holographic principles. On the other hand, a scenario in which our Universe is a four dimensional brane in a higher dimensional bulk was largely explored in literature (see Refs.[3, 4] for useful reviews on these subjects). In these models, extra-dimensions are not compactified in Calabi-Yau manifolds and the Standard Model particles are localized in the Brane. Gravitational fields are *leaked* in all the entire higher dimensional space-time. Contrary to SM particles, obtained from open strings attached in the Brane-World, gravitons are lowest massless vibrations of closed strings propagating in the bulk. This can lead to new interesting modifications of General Relativity. Brane-World cosmology seems to be particularly interesting for a fundamental understanding of the cosmo-genesis. Bounce cosmology, (Brane) inflation and phantom/quintessence dark energy may be naturally explained in these scenarios [3, 4]. The simplest one is inspired by Randall-Sundrum scenario, in which the brane-world lies in a AdS_5 -bulk [5]. RS model can be also studied using the holographic AdS/CFT conjecture [6]. RS scenario can be generalized to a Schwarzschild-De Sitter bulk and dS/CFT principle. In particular, the brane can lie in a 5D Schwarzschild-De Sitter black hole with an Hubble radius which is much larger than the brane horizon. This model may be extended considering effects of other large extra dimensions, among the $D = 9 + 1$ predicted by string theory. On the other hand, as shown by Hawking forty years ago, the evaporation of static black hole is an unavoidable consequence of euclidean semiclassical quantum gravity and quantum field theories in curved space-time [7]. In light of dS/CFT correspondence, Cardy and Verlide have shown their entropy formula in Refs.[8, 9]. This result can be generalized considering thermal fluctuations in the euclidean path integral. As is well known, the euclidean

path integral coincides with the black hole partition function, which can be considered in the macro-canonical ensemble. Mukherji and Pal in Ref.[2] have shown that in macro-canonical ensemble, the Cardy-Verlinde formula is corrected to logarithmic corrections, controlled by the black hole specific heat. As a consequence, a backreaction effect of the bulk entropy fluctuations and the brane lying inside may be expected. Nojiri, Odintsov and Ogushi (NOO) have shown that the entropy fluctuations of the bulk black hole can provide a new correction to the Hubble rate of the FRW brane. In particular, they have shown that the entropic effect is tiny in the late Brane-Universe, but it explodes in the early Brane-Universe [1] ¹. In this paper, we will analyze and generalize the Nojiri, Odintsov and Ogushi (NOO) entropic effect. In particular, we will calculate new contributions to the FRW brane equation. We will obtain new extra terms scaling as $a^{-4}Log[a^3]$, $a^{-8}Log[a^3]$. These are highly suppressed in the late Brane Universe. However, they will provide an important contribution nearby the Planck scale. This provides an interesting mechanism for a Bounce/Oscillating Cosmology in the Brane Universe.

II. LOGARITHMIC CORRECTIONS TO THE BRANE-WORLD DYNAMICS

Let us consider the five dimensional Einstein-Hilbert action

$$S = \int d^5x \sqrt{-G} \left\{ \frac{1}{16\pi G_5} R + \Lambda \right\} \quad (1)$$

We consider its Schwarzschild-De Sitter solution

$$ds_5^2 = -e^{2\rho} dt^2 + e^{-2\rho} da^2 + a^2 g_{ij} dx^i dx^j \quad (2)$$

$$e^{2\rho} = \frac{1}{a^2} \left(-\frac{a^4}{l^2} - \mu + \frac{k}{2} a^2 \right)$$

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¹ See also [10–16] for related contributions in these subjects.

l curvature radius of SdS, $\Lambda = 12/l^2$.

The thermodynamical properties of the curved SdS space-time can be evaluated from the the partition function in the gran canonical ensemble:

$$Z(\beta) = \int e^{-\beta E} \rho(E) dE \quad (3)$$

where $\rho(E)$ is the density of states, $\beta = 1/T_H$, T_H is the Hawking's temperature ($k_B = 1$). One can use the standard thermodynamical relations among the free-energy, the internal energy, the entropy and the partition function as

$$F = E - T_H S, \quad F = -T_H \text{Log}[Z] \quad (4)$$

$$e^{-\beta F} = \int dE L e^{-\beta E + S(E)}, \quad \rho(E) = L e^{S(E)} \quad (5)$$

where L is a characteristic length scale. Using the saddle point approximation, the expansion around the equilibrium point E_0 is

$$-\beta E + S(E) = -\beta E_0 + S(E_0) \quad (6)$$

$$+ \frac{1}{2} \beta^2 B(E_0) (E - E_0)^2 + O((E - E_0)^2)$$

$B(E_0)$ is a dimensionless constant. So that, one can get

$$e^{-\beta F} = \sqrt{\frac{\pi L^2}{\beta^2 B(E_0)}} e^{-\beta E_0 + S(E_0)} = e^{-\beta E_0 + S(E_0) + \frac{1}{2} \text{Log} \frac{\pi L^2}{\beta^2 B(E_0)}} \quad (7)$$

leading to

$$-\beta F = -\beta E_0 + S(E_0) + \frac{1}{2} \text{Log} \left[\frac{C(E_0) L^2}{\beta^2} \right] \quad (8)$$

where $C(E_0) = \pi/B(E_0)$. The specific heat is given by

$$\left[\frac{\partial \langle E \rangle}{\partial T} \right]_V = \beta^2 (\langle E^2 \rangle - \langle E \rangle^2) = \frac{1}{B(E_0)} \quad (9)$$

As a consequence the 0th order entropy is corrected as

$$S(E_0) = S_0 - \frac{1}{2} \text{Log} \left[\frac{C(E_0) L^2}{\beta^2} \right] \quad (10)$$

The action on the FRW brane is

$$S = \frac{W_3}{T_H} \frac{1}{2\pi G_5 l^2} \int_{a_H}^{\infty} da a^3 \quad (11)$$

renormalized as

$$S = \frac{W_3}{T_H} \frac{1}{2\pi G_5 l^2}$$

$$\times \left[\int_{a_H}^{a_{max}} da a^3 - \left(e^{\rho(a_{max}) - \rho(a_{max}, \mu=0)} \right) \int_0^{a_{max}} da a^3 \right]$$

According to Cardy-Verlinde formula [8, 9], Eq.(11) implies a leading entropy as follows

$$S_0 = -\frac{dF}{dT_H} = \frac{W_3 a_{CH,BH}^3}{4G_5} \quad (12)$$

(both cosmological horizon and black hole),

$$T_H = \frac{1}{\pi} \left(\frac{W_3}{4G_{(5)}} \right) \left(1 + \frac{2a_H}{l^2} \right) S_0^{-1/3}$$

$$F = \frac{W_3 a_{BH}^2}{16\pi G_5} \left(1 + \frac{a_{BH}^2}{l^2} \right), \quad F = -\frac{W_3 a_{CH}^2}{16\pi G_5} \left(1 + \frac{a_{CH}^2}{l^2} \right)$$

$$E = F + T_H S = \pm \frac{3W_3 \mu}{16\pi G_5}$$

with $+$ corresponding to the BH and $-$ to the CH. The Hawking's temperatures for the two horizons are

$$T_H(a_{CH}) = |de^{2\rho}/da|_{a=a_{CH}} = -\frac{1}{2\pi a_{CH}} + \frac{a_{CH}}{\pi l^2} \quad (13)$$

$$T_H(a_{BH}) = |de^{2\rho}/da|_{a=a_{BH}} = \frac{1}{2\pi a_{BH}} - \frac{a_{BH}}{\pi l^2} \quad (14)$$

where $a_{CH,BH}$ are the cosmological and black hole horizons respectively which read as

$$a_{CH,BH} = kl^2 \pm \frac{1}{2} \sqrt{k^2 l^4 - 4\mu l^2}$$

From Eq.(10), the Logharitmic correction of the Cardy-Verlinde entropy is [2]

$$S = S_0 - \frac{1}{2} \log C_v + \dots \quad (15)$$

where C_v is the specific heat

$$C_v = \frac{dE}{dT_H} = 3 \frac{2a_H^2 - l^2}{2a_H^2 + l^2} S_0$$

The 4d energy of the FRW brane in SdS bulk

$$E_4 = \frac{lE}{a} = \pm \frac{3W_3 l \mu}{16\pi G_5 a} \quad (16)$$

+ for BH, - for Cosmological. This implies that the temperature, entropy and heat capacity on the brane are related to the bulk ones as

$$T = \left(\frac{l}{a} \right) T_H, \quad S_0^B = \left(\frac{a}{l} \right)^3 S_0, \quad C_V^B = \left(\frac{a}{l} \right)^3 C_V$$

The Casimir energy corresponds to

$$E_C = 3(E_4 + pW - TS) \quad (17)$$

where $p = E_4/3W$ is the pressure where $W = a^3W_3$. Eq.(17) with uncorrected entropy is

$$E_C = \pm \left(\frac{3la^2W_3}{8\pi G_5a} \right) \quad (18)$$

The entropy shift

$$S_0 \rightarrow S_0 - \frac{1}{2} \text{Log} C_v$$

corresponds to an brane energy shift

$$\delta E_{(4)} = -\frac{T}{2} \text{Log}[C_v] \quad (19)$$

and so that to a shift in the energy density entering in the FRW brane:

$$\frac{8\pi G_4}{3} \frac{\delta E_{(4)}}{W} = -\frac{8\pi G_4}{3} \frac{T}{2W_3a^3} \text{Log}[C_v]$$

The induced metric γ on the brane

$$\gamma_{ab} = G_{\mu\nu} e_a^\mu e_b^\nu$$

where $g_{\mu\nu}$ is the Bulk metric, e_a^μ are the vectors spanning the tangent space of the brane Σ . The Parallel transports on the brane and the bulk are

$$u^a D_a u^c = 0, \quad u^\mu \nabla_\mu u^\nu = 0, \quad u^\mu = u^a e_a^\mu$$

The relations among parallel transports are

$$u^\mu \nabla_\mu u^\nu = (u^a D_a u^c) e_c^\nu - K_{ab} u^a u^b n^\nu \quad (20)$$

where K_{ab} is the extrinsic curvature of Σ and n^μ is the normal vector of Σ . Brane in the bulk is like a jump in its extrinsic curvatures K_{ab}^+, K_{ab}^- . They satisfy the junction condition:

$$K_{ab}^+ - K_{ab}^- = -\kappa_5^2 \left(S_{ab} - \frac{1}{3} S \gamma_{ab} \right)$$

where S_{ab} is the surface energy-momentum tensor of the brane. One can impose the Z_2 symmetry with respect to the surface Σ :

$$K_{ab}^+ = -K_{ab}^-$$

Form the junction condition with Z_2 symmetry with respect to Σ for the surface energy-momentum tensor, we can obtained the extended FRW equation for the brane:

$$H^2 = -\frac{1}{l^2} + \frac{k}{a^2} - \frac{8\pi G_{(4)}}{3} \rho \quad (21)$$

$$-l^2 \left(\frac{4\pi G_{(4)}}{3} \rho \right)^2 + \frac{4\pi G_{(4)} T(a)}{3W} \text{Log} C_v^B(a)$$

$$-l^2 \left(\frac{2\pi G_{(4)} T(a)}{3W} \text{Log} C_v^B(a) \right)^2$$

$$+2 \left(\frac{8\pi G_{(4)}}{3} \rho \right) \left(\frac{2\pi G_{(4)} l T(a)}{3W} \text{Log} C_v^B(a) \right)$$

with

$$\rho = \rho_{(0)} + \rho_m$$

$$\rho_{(0)} = \frac{E_4}{W} = \frac{3l\mu}{16\pi G_5 a^4}$$

$$G_{(4)} = \frac{2G_{(5)}}{l}, \quad T(a) = \frac{l}{a} T_H$$

ρ_m, ρ_Λ are matter density, localized in the FRW brane, and vacuum energy density, respectively. This equation generalizes the extended FRW equations obtained in Refs. [17–20].

In the following analysis, we will consider the case of a brane expanding in bulk and never entering in the Schwarzschild horizon.

Eq.(23) corresponds to

$$\dot{a}^2 + V(a) = k \quad (22)$$

where

$$V(a) = c_0 a^2 - c_1 a^{-2} \text{Log}[\gamma_1 a^3] + c_2 a^{-2} \quad (23)$$

$$+ c_3 a^{-6} (\text{Log}[\gamma_1 a^3])^2 + c_4 a^{-6} - c_5 a^{-6} \text{Log}[\gamma_1 a^3]$$

assuming $\rho_m = 0$.

$$c_0 = \frac{1}{l^2}, \quad c_1 = \frac{4\pi G_{(4)} l T_H}{3W_3}$$

$$c_2 = \left(\frac{8\pi G_{(4)}}{3} \right) \left(\frac{3l\mu}{16\pi G_5} \right)$$

$$c_3 = l^2 \left(\frac{\pi G_{(4)} T_H l}{3W_3} \right)^2$$

$$c_4 = l^2 \left(\frac{G_{(4)}}{3} \frac{l\mu}{2\pi G_{(5)}} \right)^2$$

$$c_5 = c_1 c_2, \quad \gamma_1 = \frac{C_V}{l^3}$$

Eq.(22) is equivalent to the one-dimensional motion of a particle. In Fig.1, we show the opportunely normalized brane potentials scanning on the space of parameters. In particular, for a brane starting from $a = \infty$, with $k = 0, 1$, and approaching $a \rightarrow 0$, it can reach a maximal value a_{max} and return back, re-expanding. On the other hand, for a brane starting from $a = 0$, it can reach the maximum a_{max} and it can re-collapse back to $a = 0$.

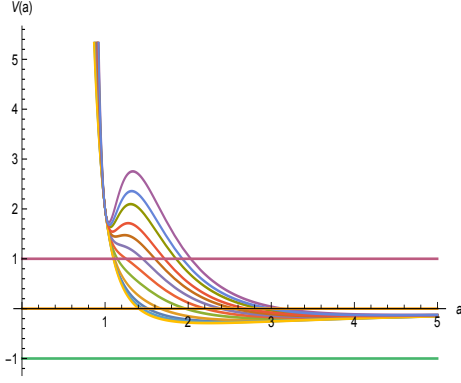


FIG. 1. The Brane potentials $V \equiv V(a)$ are displayed with several choices of the parameters (the unit convention $c_{1,2,4,5}\gamma_1 = 1$ varying c_3). Brane turning-points levels $k = -1, 0, 1$ are also displayed.

III. TOWARD A REALISTIC EKPYROTIC BRANE COSMOLOGY

In the Bounce scenario triggered by the NOO effect, the growing of anisotropies in the energy-momentum tensor are expected to be order $O(1)$. This is a typical problem in every Bounce mechanisms: the new Universe would emerge from the BLK (Belinsky, E. M. Lifshits and I. M. Khalatnikov) singularity from the pre-Universe [21–23]. Of course this problem can be solved assuming a successive inflation mechanism. For example a brane inflation scenarios could over-implemented in our picture. However, in this section, we will consider another solution involving the introduction of an extra exotic super-stiff fluids with $p_\phi > \rho_\phi$ [24, 25]. As is known, a way-out from the BLK anisotropies is that the Early Universe is dominated by the overall condition $p_\phi > \rho_\phi$ [24]. However, in our case, the introduction of the exotic superstiff fluid introduce a new interesting term $\rho_\phi a^{-4} \text{Log}[a^{-3}]$. Such a term is repulsive in the late Universe, while attractive in the very Early Universe. An example of superstiff fluid is a scalar field with an exponential potential:

$$L_\phi = \frac{1}{2} \partial_\mu \phi \partial^\mu \phi - V(\phi), \quad V(\phi) = -V_0 e^{\phi/M} \quad (24)$$

The corresponding energy density is positive and highly increasing with a decreasing:

$$\rho_\phi = \frac{1}{2} \dot{\phi}^2 + V(\phi) = \frac{6M^2\alpha}{t^2} = \frac{6M^2\alpha}{a^{2/\alpha}} \quad (25)$$

while the pressure is

$$p_\phi = \frac{1}{2} \dot{\phi}^2 - V(\phi) = \frac{4M^2}{t^2} = \frac{4M^2\alpha}{a^{2/\alpha}} \quad (26)$$

where $a(t) = |t|^\alpha$ and $w > 1$ corresponds to $\alpha < 1/3$. The presence of such a fluid generates an repulsive entropic term contributing to as H^2 as

$$2 \left(\frac{8\pi G_4}{3} \rho_\phi \right) \left(\frac{2G_4}{3Wl} \text{Log} 3S_0 \right) \sim a^{2/\alpha} a^{-4} \text{Log}[(\dots)a^3] \quad (27)$$

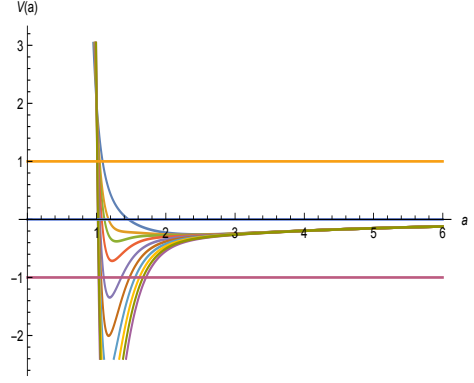


FIG. 2. The Brane potentials $V \equiv V(a)$ in presence of superstiff fluids are displayed. We chose $\alpha = 1/4$, with several choices of the parameters. (the unit convention $c_{1,2,3,4,5}, \gamma_1 = 1$ varying the overall coefficient in Eq.(27). Brane turning-points levels $k = -1, 0, 1$ are also displayed.

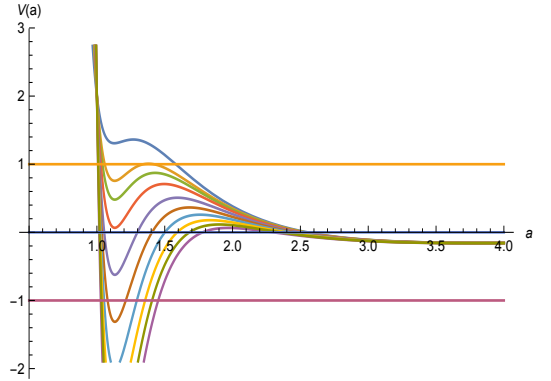


FIG. 3. The Brane potentials $V \equiv V(a)$ in presence of superstiff fluids are displayed. Brane turning-points levels $k = -1, 0, 1$ are also displayed.

which is minor than $< a^{-9} \text{Log}[(\dots)a]$. In Fig.2-3, we show the effect of such an entropic term with the growing of M/M_{Pl} in the case $\alpha = 1/4$ and $\mu = 1$. As we can see, the new term generated a potential barrier which radically changes the bounce dynamics. As an alternative, we consider a repulsive superstiff exotic fluid, with the same potential of Eq.(24). In Fig.4, the brane dynamics in presence of a repulsive superstiff fluid is displayed.

IV. CONCLUSIONS

In this paper, we have shown how the thermal entropy fluctuations of a higher dimensional Schwarzschild-De Sitter black hole may provoke modifications of the Brane FRW dynamics. We have generalized the Nojiri-Odintsov-Ogushi result, calculating a more complete equation for the brane lying in the Schwarzschild-De Sitter black hole. In particular, neq entropic terms are one scaling as $a^{-4} \text{Log}[a^3]$ and $a^{-8} (\text{Log}[a^3])$. The first one can be interpreted as a *repulsive dark radiation* in the Late

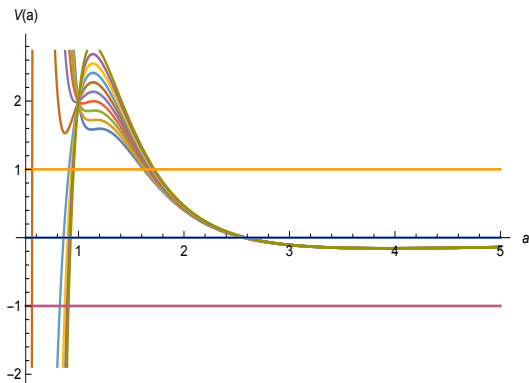


FIG. 4. The Brane potentials $V \equiv V(a)$ in presence of superstiff fluids are displayed. Brane turning-points levels $k = -1, 0, 1$ are also displayed.

Universe, while becoming attractive in the Early Universe $a \rightarrow 0$. The second class of terms provides highly suppressed contributions in the Late Universe, while exploding for $a \rightarrow 0$.

This implies that entropic terms may relevantly change the dynamical behavior of the brane in the early Universe and in the very late Universe.

On the other hand, we also find terms scaling as $\rho_m a^{-4} \text{Log}[a^3]$ where ρ_m is the matter density localized in the four dimensional brane. As a consequence, the fate of the brane Universe is highly dependent by the matter and vacuum energy content in it and in the bulk. These new matter/vacuum dependent terms are fundamentally important in realistic Ekpyrotic Brane Cosmology. As a useful example, we have shown the case of a superstiff fluid which washes out BLK anisotropies in the bounce. In this case, the superstiff fluid also triggers the Bounce mechanism thanks to the associated extra entropic repulsive term. As a consequence, the superstiff fluid does not

only solve the BLK anisotropy problem but it is also a source of the Bounce super-repulsion.

Let us also comment on the possible presence of cosmological terms induced by dark Yang-Mills condensates or dark non-linear Born-Infeld electrodynamics. In the framework of dark Yang-Mills or dark QCD condensates, cold dark matter and dark energy can be elegantly unify in a common framework, as recently shown in Refs.[28, 29]. On the other hand, if the Dark Born-Infeld sector is coupled to neutrinos, a order 10^{-3} eV neutrino mass can be generated and connected with dark energy [30]. In both these scenarios the repulsive condensates may change in cosmological time and their associated entropic terms may drive the early cosmological evolution in a complicated way. A complete analysis of these cases deserves further investigations beyond the purposes of this letter. Finally, let us note that the entropic corrections may imply interesting unexplored effects in more complicated Brane-world scenarios. For example in $f(R)$ -Brane worlds, interesting amplifications or suppressions of the entropic terms would be obtained. On the other, in Extended Theories of gravity, the thermodynamical behavior of black hole may be highly non-trivial. For instance, as discussed in Ref.[31], the Hawking's emission is exponentially suppressed for Bousso-Hawking-Nojiri-Odintsov antievaporating solution [32–34]².

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